The effect of the neutron star crust on the evolution of a core magnetic field

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February 1, 2008

Abstract

We consider the expulsion of the magnetic field from the super–conducting core of a neutron star and its subsequent decay in the crust. Particular attention is paid to a strong feedback of the distortion of magnetic field lines in the crust on the expulsion of the flux from the core. This causes a considerable delay of the core flux expulsion if the initial field strength is larger than 10^{11} G. It is shown that the hypothesis on the magnetic field expulsion induced by the neutron star spin–down is adequate only for a relatively weak initial magnetic field $B \approx 10^{11}$ G. The expulsion time–scale depends not only on the conductivity of the crust, but also on the initial magnetic field strength itself. Our model of the field evolution naturally explains the existence of the residual magnetic field of neutron stars. Its strength is correlated with the impurity concentration in neutron star crusts and anti–correlated with the initial field strengths.

Key words: magnetic fields - stars: neutron - pulsars: general - stars: evolution

1 Introduction

Where the neutron star magnetic field (MF) is located, by which processes it is generated and what determines its strength, structure and evolution, is the subject of scientific debates since neutron stars are conceivable for human mind. Regarding the field location there exist basically two qualitatively different ideas: the field is either present in the entire star, that is, in the core and the crust, or is located mainly in the crust. Whether the entire neutron star or only the crust is penetrated by the field depends strongly on the mechanism responsible for its generation.

Both the simple idea of flux conservation during the gravitational collapse and the action of a dynamo in the convective proto–neutron star (Thompson & Duncan 1993) result in a field structure which penetrates the entire star. On the other hand it is possible that fall–back accretion after the supernova submerges any initially existent field (see Geppert, Page & Zannias 1999). Since the re–diffusion process may last a long time, the existence of young pulsars (PSRs) could be explained - at least to some extent - by the action of a thermoelectric instability (Blandford, Applegate & Hernquist 1983, Urpin, Levshakov & Yakovlev 1986, Wiebicke & Geppert 1996). A strong MF can be produced rather rapidly in the surface layer of the neutron star by transforming heat flux into magnetic flux. If so, the surface MF which governs e.g. the spin–down of the PSR is maintained by currents confined in the crust alone. However, the currently existing models describe only the amplification of large scale toroidal fields, the production of the observed dipolar MF is still an open problem.

Although the assumption of a purely crustal MF explains well the observed long–term evolution of isolated neutron stars (Urpin & Konenkov 1997) and of neutron stars accreting in binary systems (Konar & Bhattacharya 1997, Urpin, Geppert & Konenkov 1998, Urpin, Konenkov & Geppert 1998) the evolution and effect of a core MF have to be investigated too. The existence of a core MF may explain transient features like the post–glitch behaviour (Alpar, Langer & Sauls 1984, Chau, Cheng & Ding, 1992) as well as the presence of residual surface fields in old neutron stars (e.g. Jahan Miri & Bhattacharya 1994).

It is generally accepted that a phase transition into the super–fluid/super–conductive state takes place in the core rather early in a neutron star's life if the temperature drops below 10^{10} K (see Alpar 1991, Page 1998). Given the typical neutron star MF strength, the core behaves as a type II superconductor (Baym, Pethick & Pines, 1969), i.e. the MF penetrates the core in quantized flux tubes (fluxoids). The evolution of the core field is connected with movements of the fluxoids caused by forces acting upon them. Under the action of these forces the fluxoids move towards the core–crust boundary, the magnetic flux is expelled from the core into the crust, where the magnetic field suffers Ohmic decay.

Ding, Cheng & Chau 1993 (hereafter DCC) described these forces and developed a method to calculate the radial velocity of the fluxoids, which in turn determines the rate of flux expulsion from the core. There are the buoyancy force (Muslimov & Tsygan 1985), the drag force (Harvey, Ruderman & Shaham, 1986) and the force exerted by the neutron

vortices which act upon the fluxoids. The interaction between vortices and fluxoids is determined by their pinning energy; in the course of neutron star spin–down the vortices move outward and fluxoids can move faster, comove or move slower than vortices.

However, the motion of the proton flux tubes in the core leads to a distortion of the field structure in the crust near the crust–core boundary. This changes the magnetic energy in the crust which, in turn, is the source of a force that influences the flux expulsion. The aim of this paper is to consider that "back reaction" of the crust onto the fluxoid movement.

The paper is organized as follows. In section 2 we describe the forces acting on the fluxoids in the core, the procedure of self–consistent determination of the inner boundary condition for the crustal field decay, and how to find the surface and core MF as functions of time. Section 3 represents the numerical results and in section 4 we discuss the consequences of crustal effects for the long term evolution of isolated neutron stars and compare our results with those of other authors.

2 Description of the model

As shown by Alpar, Langer & Sauls (1984) (see also DCC) both the proton fluxoids and the neutron vortices are associated with large magnetic fields of comparable strengths. Interaction of both structures involves a pinning energy of $E_p \sim 10$ MeV per intersection, and the strongest pinning occurs in the core regions where the flow of vortices is nearly perpendicular to the fluxoids.

The flow velocity of the vortices, v_n, is governed by the spin–down torque, which in a steady state is assumed to be determined by the magnetic dipole braking of the neutron star (see DCC):

$$v_n(t) = \frac{rk(t)\Omega_s^2(t)}{2} \quad , \tag{1}$$

where

$$k(t) = \frac{8B_e^2(t)R^6 \sin^2\chi}{3Ic^3} \quad . \tag{2}$$

Here, Ω_s is the angular velocity of the core super–fluid. Its difference to the observed rotation rate of the crust, Ω_c, causes the driving Magnus force; R is the radius of the neutron star, χ is the angle between the rotational and the magnetic axis of the neutron star, I its moment of inertia and B_e is the surface magnetic field in the equatorial plane. We will assume that equation (1) holds for all distances from the rotation axis $r \leq R_c$, where R_c is the radius of the super–conducting core.

With respect to the relation between the radial velocity of the proton fluxoids, v_p, (for justifications to neglect their azimuthal motion see discussion in DCC) and v_n one can distinguish three different regimes. Either the vortices cross the fluxoids (*forward*

creeping), or both types of tubes move with the same velocity (*comoving*), or the fluxoids move even faster than the vortices (*reverse creeping*). Thus, in case of forward creeping the vortices exert a force onto the fluxoids which drives them outward, while in case of reverse creeping a resistive force acts upon the fluxoids which counteracts the flux expulsion. It is clear that this vortex acting force f_n depends on the angular velocity lag $\omega = \Omega_s - \Omega_c$ and changes its direction, when the Magnus force changes its sign because the core super–fluid rotates slower than the crust and the charged components of the core coupled with it. The maximum lag that can be sustained by the pinning force, $\omega_{cr} \propto E_p$, defines also the maximum force a vortex can exert onto a fluxoid. This force per unit length of the fluxoid is given by (DCC, Chau, Cheng & Ding, 1993)

$$f_n = \frac{2\Phi_0 \rho r \Omega_s(t)\omega(t)}{B_c(t)} \quad , \tag{3}$$

where Φ_0 denotes the quantized flux per fluxoid, $\Phi_0 = hc/2e = 2 \cdot 10^{-7}$ G$\cdot$cm^2, ρ is the density of the matter and B_c is the mean strength of the core MF, proportional to the number of fluxoids in the core N_p and given by $B_c = \Phi_0 N_p / \pi R_c^2$.

Independently of the rotational evolution of the neutron star the fluxoids are affected by a buoyancy force f_b, which is caused by the magnetic stress at the surface of the fluxoid. This force per unit length of the fluxoids is given by (Muslimov & Tsygan, 1985):

$$f_b = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \frac{1}{R_c} \ln\left(\frac{\lambda}{\xi}\right) \quad . \tag{4}$$

The relation between the London penetration length λ and the coherence length ξ defines the condition for the formation of a type II superconductor, it requires that $\xi < \lambda/\sqrt{2}$.

The drag force is caused by the scattering of the degenerate ultra–relativistic electrons at the fluxoid MF. In an approximation which neglects collective effects of the fluxoids, the drag force per unit length is proportional to v_p and is given by (Harvey, Ruderman & Shaham, 1986):

$$f_v = -\frac{3\pi}{64} \frac{n_e e^2 \Phi_0^2}{E_F \lambda} \frac{v_p}{c} \quad , \tag{5}$$

where n_e is the number density of the electrons in the core, about 5% of the neutron number density, and E_F is the Fermi energy of the electrons. For the Fermi energy as well as for λ and ξ we take the values determined by the density at the crust–core interface, ρ_c. Collective effects which modify the drag force become important only for $B_c \geq 10^{15}$ G, when the mean field becomes comparable to the field inside the fluxoids.

Additionally, DCC take into account tension forces which result in a coefficient of f_b of the order of unity in almost the entire star. For simplification we will assume that the fluxoids are not bended but parallel to each other while moving together with their roots. According to DCC, the velocity of the fluxoids is derived from $f_n + f_b + f_v(v_p) = 0$.

However, the MF which is concentrated in flux tubes in the core, penetrates the crust too. The movement of the roots of the fluxoids leads to a bending of the magnetic field

lines in the crust (see figure 1). The power done by the core forces is equal to the net Poynting flux through the core surface, which, in turn, is equal to the rate of change of the sum of mechanical and field energy in the volume outside the core. We will assume the crust to be crystallized so that the change of mechanical energy corresponds only to the Joule heating of the crust. The contribution of the electric field to the field energy can certainly be neglected. Thus, we find for the balance of powers:

$$\sum_{\text{fluxoids}} \int (f_n + f_b + f_v) v_p \, \mathrm{d}l = \int\limits_{V_{\text{crust}}} \frac{j^2}{\sigma} \, \mathrm{d}V + \frac{\mathrm{d}}{\mathrm{d}t} \int\limits_{V} \frac{B^2}{8\pi} \, \mathrm{d}V \; . \tag{6}$$

Note, that f_v depends on v_p as well as the r.h.s. of equation (6). In the l.h.s. the integration is performed over a fluxoid's length, the summation runs over the number of fluxoids. For sake of simplicity we consider all terms of the l.h.s. of equation (6) to be position independent, take them at $r = R_c$ and approximate that side by $(f_n + f_b + f_v) N_p \langle l \rangle v_p$. Since the length of a fluxoid is $l = 2\sqrt{R_c^2 - r^2}$, its mean value is $\langle l \rangle = 4R_c/3$. The number of fluxoids decreases with core field decay. The multiplication with the fluxoid velocity v_p yields the core forces power which has to be balanced by Joule heating and the change of the magnetic energy. While the Joule heating is restricted to the volume of the crust, the change of the magnetic energy has to be calculated both for the crust and the vacuum environment of the neutron star. We introduce the core forces

$$F_{n,b,v} = f_{n,b,v} \cdot 4R_c/3 \cdot N_p \tag{7}$$

and the crustal force

$$F_{crust} = -\frac{1}{v_p} \left(\int\limits_{V_{\text{crust}}} \frac{j^2}{\sigma} \, \mathrm{d}V + \frac{\mathrm{d}}{\mathrm{d}t} \int\limits_{V} \frac{B^2}{8\pi} \, \mathrm{d}V \right) . \tag{8}$$

Thus, equation (6) can be rewritten in the form $F_n + F_b + F_v + F_{crust} = 0$.

In this paper we consider the evolution of a purely poloidal dipolar MF. Hence, the appropriate representation for the MF is that of the Stokes stream function $S(r,t)$ which is related to the vector potential $\vec{A} = (0,\, 0,\, A_\varphi)$ with $A_\varphi = S(r,t)\sin\theta/r$, where r and θ are the spherical radius and polar angle, respectively. Then, we can express the spherical field components, B_r and B_θ, in terms of $S(r,t)$,

$$B_r = \frac{2S}{r^2}\cos\theta \, , \;\; B_\theta = -\frac{\sin\theta}{r} \cdot \frac{\partial S}{\partial r}. \tag{9}$$

We normalize $S(r,t)$ by its initial surface value at the equator, $S(R,0) = B_{e0}R^2$, $s(r,t) = S(r,t)/S(R,0)$, so that $B_e(t) = B_{e0} \cdot s(R,t)$.

While in the core there is no ohmic decay but motion of the fluxoids, ohmic diffusion determines the field evolution in the solid crust. Thus, the field evolution in the core $(r < R_c)$ is governed by

$$\frac{\partial s}{\partial t} = -v_p \, \frac{\partial s}{\partial r}, \tag{10}$$

but in the crust ($R_c < r < R$) by

$$\frac{\partial s}{\partial t} = \frac{c^2}{4\pi\sigma} \left(\frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2} \right). \tag{11}$$

Therefore, we find for the r.h.s. of equation (6) the following expressions:

$$\int\limits_{V_{\rm crust}} \frac{j^2}{\sigma} \, {\rm d}V = \frac{c^2 B_{e0}^2 R^4}{6\pi} \int\limits_{R_c}^{R} \frac{1}{\sigma} \left(\frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2} \right)^2 {\rm d}r, \tag{12}$$

$$\frac{\rm d}{{\rm d}t} \int\limits_{V_{\rm crust}} \frac{B^2}{8\pi} \, {\rm d}V = B_{e0}^2 R^4 \, \frac{\rm d}{{\rm d}t} \left(\frac{2}{3} \int\limits_{R_c}^{R} \frac{s^2}{r^2} \, {\rm d}r + \frac{1}{3} \int\limits_{R_c}^{R} \left(\frac{\partial s}{\partial r} \right)^2 {\rm d}r \right), \tag{13}$$

$$\frac{\rm d}{{\rm d}t} \int\limits_{V_{\rm vacuum}} \frac{B^2}{8\pi} \, {\rm d}V = \frac{2 B_{e0}^2 R^3}{3} \; s(R,t) \; \frac{{\rm d}s(R,t)}{{\rm d}t}. \tag{14}$$

The assumption of a homogeneous mean MF in the core leads to the following expression for the stream function there

$$s(r,t) = \frac{B_c(t)}{2} \, r^2. \tag{15}$$

The electric conductivity in the solid crust consists mainly of contributions from electron–phonon and electron–impurity scattering. Electron–phonon interactions dominate the transport at high temperatures and relatively low densities, whereas the impurity concentration determines the conductivity at lower temperatures and larger densities. We use the numerical data for the phonon conductivity obtained by Itoh, Hayashi & Koyama (1993) and an analytical expression for the impurity conductivity derived by Yakovlev & Urpin (1980). The crustal temperature which influences the electron–phonon conductivity is taken from cooling curves calculated for different neutron star models by Van Riper (1991). For the chemical composition we adopt that of cold catalyzed matter. Note that the longterm crustal field decay in isolated neutron stars is determined by the impurity concentration mainly which is characterized by the impurity parameter Q because after $\approx 10^6$ years the neutron star cools down completely. Moreover, the currents generated by flux expulsion from the core in the crust are located in its deep high density regions. Thus, this field evolution is almost insensitive to the cooling history of the neutron star. The assumed homogeneity of the core MF leads to following ansatz for the fluxoid velocity:

$$v_p = \alpha(t) r. \tag{16}$$

Thus, the solution of equation (10) is

$$s(r,t) = s(r,0)\exp\left(-2\alpha(t)t\right), \tag{17}$$

where $\alpha(t)$ has to be determined by the self–consistent solution of equation (6). In that way, v_p determines the inner boundary condition for equation (11).

Since the MF in the center of the neutron star has to be regular we demand as the inner boundary condition that $s(r,t)/r^2$ remains finite with $r \to 0$, which is fulfilled by our choice of $s(r,t)$ in the core. The boundary condition at the neutron star surface has to ensure a matching of the interior field with the external dipolar field. That is expressed by the requirement $R\frac{\partial s}{\partial r} = -s$ at $r = R$.

When solving equation (11) the boundary condition at the crust–core boundary is given by equation (17) with $s(R_c,t)$, which decreases generally non–exponentially because α depends on time.

In order to solve equation (6) we follow the ideas of DCC, however, we have to consider additionally the crustal effects given by the r.h.s. of the equation (6). By the choice of the strength of the core MF, a profile of $s(r,0)$ in the crust, an initial rotational period and a certain EOS which determines mass, radius and moment of inertia of the neutron star, we define the values of the parameters at $t = 0$.

Then we start with $v_p = v_n$ or $\alpha(t) = k(t)\Omega^2/2$ where $k(t)$ is given by equation (2). Adopting that equality we calculate $s(R_c,t)$ from equation (17) which represents the inner boundary condition for equation (11). By use of equations (12), (13) and (14), we calculate the r.h.s. of equation (6), or F_{crust}. After calculating F_b and F_v, finally, we obtain F_n and thus ω. The core MF defines also the maximum lag ω_{cr} that can be sustained by the pinning force (see equation (11) in DCC), which gives an upper limit of F_n. If $-\omega_{cr} < \omega < \omega_{cr}$ then the fluxoids and vortices comove and v_p, found by the procedure described above, is the real velocity of the fluxoids.

If $\omega > \omega_{cr}$ then the fluxoids are in the forward creeping regime. Since ω_{cr} defines the maximum force which can be exerted onto the fluxoid we set $\omega = \omega_{cr}$ and calculate $F_n(\omega_{cr})$. The solution of equation (6) with $F_n(\omega_{cr})$ provides v_p. At the forward creeping stage $v_p < v_n$, thus v_p will be found as a root of equation (6) in the interval $[0, v_n]$ using the bisectional method. If $\omega < -\omega_{cr}$ then the reverse creeping mode is reached. In this case we set $\omega = -\omega_{cr}$, calculate $F_n(-\omega_{cr})$ and solve again equation (6). This yields $v_p > v_n$.

After having determined the velocity of the fluxoids in that way we use equations (11) and (17) to calculate the evolution of $s(r,t)$ and to obtain the temporal behaviour of the MF at the crust–core boundary B_c and at the neutron star surface B_e. From the latter we deduce the evolution of the spin period $P(t) = 2\pi/\Omega$. The described set of computations is performed at each time step.

3 Numerical results

Although the induction equation is linear, the problem is a nonlinear one in terms of the magnetic field. Additionally, equation (6) which determines the flux expulsion velocity

is nonlinear in v_p too. Thus, we have to consider the time evolution for different sets of initial values of B_e and B_c and to investigate their temporal evolution.

For the calculations below we use the standard cooling scenario considered by Van Riper 1991 for the neutron star model based on the Friedman–Pandharipande (FP) equation of state (Friedman & Pandharipande 1981) with the total mass $M = 1.4 M_\odot$, radius of the star $R = 10.61$ km and radius of the core $R_c = 9.67$ km.

In the present paper we consider the case that the homogeneous core MF has initially the same strength as the surface MF (see curve 1 in figures 1, 2). At first we assume for both B_e and B_c the standard values 10^{12} G and for the initial spin period 10 ms. As described above, the expulsion of the core MF causes distortions of the crustal field in the vicinity of the core–crust boundary (see figures 1, 2), generating currents just in that region. Hence, the decay of the crustal field, characterized by the decay time of the surface MF τ_s, is almost completely determined by the impurity parameter Q. The larger Q the more rapidly the crustal field may decay which, in turn, makes the braking of neutron star rotation by magneto–dipole radiation less efficient. If the MF of the neutron star is confined in the crust only $\tau_s \sim 10^6\, Q^{-1}$ years is the appropriate estimate for the chosen neutron star model (Urpin & Konenkov 1997). However, the existence of a core MF and its expulsion into the crust causes a nonlinear dependence of the inner boundary condition for equation (11) on the MF evolution and deviations from the above estimate for τ_s will occur.

In figure 3 we show from top to bottom the evolution of the MF, of the velocities of both types of flux tubes, of the forces acting upon the fluxoids, and of the rotational period of the neutron star, calculated by the procedure described in the section above. Calculations were performed for three values of the impurity parameter $Q = 1$ (left column), $Q = 0.1$ (middle column) and $Q = 0.01$ (right column).

Almost independently on Q, the velocity of the fluxoids at times $t < 10^{4.5}$ is determined by the balance of the vortex acting force F_n and the drag force F_v. Since the spin–down of the neutron star is very effective, the vortices move outward fastly thereby cutting through the fluxoids: they are in the forward creeping regime. At this stage v_p falls down from 10^{-7} to about 10^{-8} cm/s. However, the characteristic time of expulsion $\tau_e = R_c/v_p \approx 10^6$ years, and the core MF remains almost constant. Nevertheless, even during that relatively short period a strong meridional field component B_θ is generated in the crust close to the crust–core interface. This gives raise of the crustal force F_{crust}, and later v_p is determined by the balance of crustal and buoyancy forces. The power–like decay of F_n simply reflects the increase of the spin period of the neutron star $P \propto \sqrt{t}$ with an almost constant surface MF and the subsequent decrease of the vortex number density in the core. At this stage, v_p is dependent on Q: the lower Q the lower v_p, an effect of the crustal force, counteracting the expulsion. As long as the fluxoids are in the forward creeping regime, B_c remains nearly constant. The forward creeping regime is followed by the comoving regime during which F_n changes its sign, and vortices and fluxoids move with the same velocity. Until that moment, the flux expulsion rate is governed by the balance of crustal

and buoyant forces, however, the crustal force becomes ineffective on larger time–scales, $t > \tau_e$. This is the reason for the increase of v_p and, hence, the decrease of B_c. Note, that the evolution of B_e follows closely that of B_c. The expulsion lasts till another force can prevent it. It appears to be a common situation that at late evolutionary stages the velocity of the fluxoids is determined by the balance of buoyancy and vortex acting forces. The amount of flux which has to be expelled from the core in order to reach such a balance is easy to predict by comparison of F_n and F_b just before the beginning of the expulsion. Since $F_b \propto B_c$ but $F_n \propto \sqrt{B_c}$, the residual MF strength can be estimated by

$$\frac{B_{res}}{B_{e0}} = \left(\frac{F_n(\tau_e)}{F_b(\tau_e)}\right)^2 \quad . \tag{18}$$

The longer the time–scale of expulsion (due to the resistive effect of the crustal force), the lower is the residual field strength. This is because the number of vortices and F_n are lower if the neutron star spins down to a greater rotational period during a longer time. Thus, F_n will be able to balance F_b on a lower MF level. In turn, the time–scale of expulsion depends on the impurity concentration. Namely, for $Q = 1,\ 0.1,\ 0.01$ the residual field strength is about $10^{10},\ 10^9,\ 10^8$ G, while τ_e is about $10^7,\ 10^8,\ 10^9$ years, respectively. The velocity of the fluxoids increases just after the beginning of the reverse creeping stage, and decreases sharply when F_b balances F_n. Concluding, we want to point out that the main force which expels the fluxoids from the core is the buoyancy force. The vortex acting force influences the evolution of the MF of very young and very old neutron stars, while the effect of the drag force is restricted to the very early stage only.
In figure 4 we show the same quantities as in figure 3 but the initial strength of the core and surface MF is assumed to be only 10^{11} G. This leads to a drastically less efficient spin–down of the neutron star. Because the magnetic energy stored in the crust and in the surrounding vacuum is $\propto B_e^2$, the influence of the crustal forces on the expulsion is much weaker than in the case of $B_{e0} = 10^{12}$ G. For a comparatively low field strength the situation is similar to that considered by DCC because practically only F_b, F_n and F_v determine the field evolution. Due to the much slower spin–down, from the beginning both vortices and fluxoids are moved with the same velocity. Again, in the young neutron star ($t < 10^6$ years), F_n and F_v are much greater than F_{crust} and F_b, while in the old neutron star F_n and F_b are the dominant forces. Since the crustal force is much weaker, $\tau_e << \tau_s$, i.e. the evolution of B_c is decoupled from that of B_e.
The evolution of core and surface MF and of the spin period for a strong initial MF, $B_{e0} = 10^{13}$ G, is shown in figure 5. In this case the strong crustal force prevents the flux expulsion from the core much more effectively. The strong and long living MF causes an efficient spin–down, up to $30 - 300$s, depending on the impurity concentration. The residual field strength is about $3 \cdot 10^7$ G for $Q = 1$, and about $3 \cdot 10^6$ G for $Q = 0.1$. For even lower impurity concentrations τ_e exceeds the Hubble time. Note that the large initial MF enforces $\tau_e = \tau_s$. This is a consequence of the strong crustal force, which decelerates effectively the expulsion and couples the evolution of the core with that surface MF.

Generally, the residual field strength is anti–correlated with the initial field strength and positively correlated with Q. Once the balance of crustal and buoyant forces is replaced by the balance of vortex acting and buoyant forces on a much lower level, both B_c and B_e decay down to a residual value, determined by the final rotational period P_f of the neutron star. For such neutron stars which do not reach their P_f even after 10^{10} years (as for $B_{e0} = 10^{13}$ G, $Q = 0.01$ and $B_{e0} = 10^{11}$ G, $Q = 1$) a residual MF is not attained.
Among the processes considered here are two dissipative ones: the work done by the drag force and the Joule heating produced by field decay in the crust. Assuming that all heat produced in that way is irradiated from the surface one can estimate the corresponding surface temperature T_s by use of the relation

$$\dot{Q} = 4\pi R^2 \sigma_{SB} T_s^4 \ , \tag{19}$$

where σ_{SB} is the Stephan–Boltzmann constant, $\dot{Q} = F_v \cdot v_P + \dot{Q}_{Joule}$, and $\dot{Q}_{Joule}$ is given by equation (12) (see Miralles et al. 1998 for the discussion of the validity of this equation). In figure 6 the temporal evolution of T_s is presented for different initial MF strengths and the values of the impurity parameter considered above. When the standard cooling scenario for a FP–neutron star applies it becomes clear that the contribution of those dissipative processes is considerable during the photon cooling era in relatively old ($t > 10^6$ years) radio–pulsars.

4 Discussion

We considered the effect of the neutron star crust onto the expulsion of a core MF and its ohmic decay in the crust. To this aim we solved self–consistently the equation of balance of the powers of forces acting on the fluxoids in the core and the rate of change of magnetic energy outside the core of the neutron star, assuming a homogeneous MF in the core for all the life of the neutron star.
The evolution of B_e determines the rotational evolution of the neutron star, which in turn has considerable effects on the vortex acting force defining the residual field strength.
It turns out that the characteristic time–scale of the decay of the surface MF τ_s increases with increasing initial MF strength and decreasing impurity parameter. The amount of B_e–decay at $t \geq \tau_s$ is correlated with the spin–down rate. A slow spin–down to $P_f \approx 1$ s results in a small field decay by less than two orders of magnitude ($B_{e0} = 10^{11}$ G, $Q = 0.01$) while a drastic spin–down to $P_f \approx 300$ s yields a field decay by about seven orders of magnitude ($B_{e0} = 10^{13}$ G, $Q = 0.1$).
Comparing our results with those obtained for a purely crustal field decay in isolated neutron stars (Urpin & Konenkov 1997), we find a considerable deceleration of the decay of a field penetrating the entire star. The field evolution for the FP model with standard cooling and $Q = 0.01$ in the case of crustal MF yields an impurity dominated decay with $\tau_s \approx 10^8$ years which becomes then nearly exponential. In the present model, only for the small initial field of 10^{11} G, $\tau_s \approx 10^8$ years, while it is 10^9 years for $B_{e0} = 10^{12}$ G and is in

the order of the Hubble time scale for $B_{e0} = 10^{13}$ G. Evidently, for a purely crustal MF with the inner boundary condition $s(R_c, t) = 0$ valid for all neutron star's life, no residual field can be obtained. The existence of a residual field as well as the strong dependency of τ_s on B_{e0} reflects the nonlinear mutual dependency of the field and rotational evolution, governed by the balance of forces acting upon the fluxoids.

Bhattacharya & Datta, 1996, studied the decay of a neutron star MF just expelled from the core and deposited in the bottom layers of the crust. They found a rather strong decrease of the final MF strength after 10^{10} years with increasing impurity parameter. This result can not be confirmed by our investigation: *a larger Q results in a higher residual field*. This is due to the fact that for a larger Q the crustal force is less strong, the time–scale of expulsion is shorter, the surface field decays faster, the spin–down is less efficient and the residual field determined by balance of the F_n and F_b is stronger.

Taking into account that under the assumption of a purely crustal MF a value of $Q = 0.1...0.01$ is at least not in disagreement with observations (Urpin & Konenkov 1997), these values taken for our model result in a constant B_e for almost all conceivable pulsar lifetimes; even for $B_{e0} = 10^{11}$ G and $Q = 1$ a remarkable field decay would start only for $t > 10^6$ years. Note that the early ($t \leq 10^6$ years) cooling–determined crustal field evolution is included in our investigations. Only extremely large impurity parameters (perhaps $Q > 1$, which would reflect qualitative deviations from the bcc crystalline structure of the crust) would allow for a field decay in $t < 10^6$ years. Thus assuming an initial field strength larger than 10^{11} G, our study yields a $\tau_s > 10^7$ years, in agreement with the statistical results for isolated radio–pulsars found by Bhattacharya et al. (1992) and Hartman et al. (1996).

It is clearly seen from figure 3, that the main force which is responsible for expulsion of the flux from the core is the buoyancy force. The hypothesis about the so called "spin–down induced" expulsion of magnetic flux (Konar, Bhattacharya 1998 and references therein) seems to be adequate only in case of a weak ($\sim 10^{11}$ G) initial magnetic field. If the magnetic field is stronger, say, 10^{12} G, vortices will cut through the fluxoids, whereas the latter are anchored in the crust for the time of $\geq 10^7$ years even in case of high impurity concentration ($Q = 1$). During this time the neutron star spins down by a factor of about 100, while B_c remains almost the same, in contradiction with the core field evolution predicted by the spin–down induced mechanism of expulsion.

In the present paper we study the magneto–rotational evolution of isolated neutron stars suffering a spin–down by magneto–dipole radiation. We found that for relatively large field strengths, $10^{12}\text{G} \leq B_{e0} \leq 10^{13}\text{G}$, the residual field $B_{\rm res} \propto P_f^{-2}$, in accordance with the result of DCC. We also found that in this range of B_{e0} the expulsion times–scale is determined mainly by the balance of buoyant and crustal forces. These forces are independent of the spin–down rate of the neutron star, i.e. the expulsion time–scale does not depend on the specific braking mechanism. Jahan Miri & Bhattacharya, 1994, studied the evolution of neutron stars in binaries. They adopted the hypothesis on the spin–down induced magnetic flux expulsion and found $B_{\rm res} \propto P_{max}^{-1}$, where P_{max} is the maximum

period reached by the neutron star during the propeller phase. We do not expect that our model (if applied to the neutron star evolution in binaries) will confirm this result.
In the context of the current discussion of magnetars, which are thought to be highly magnetized isolated neutron stars spinning down by magneto–dipole braking, the investigation of the crustal effect onto the expulsion of initial MFs $> 10^{14}$G is an urgent task. Our present model is not suitable for that purpose since we consider only ohmic diffusion in the crust. In the case of extremely large field strength as expected for magnetars, the field evolution in the crust is more complicated, effectively accelerated by a Hall cascade (Goldreich & Reisenegger 1992) and/or fracturing of the crust (Thompson & Duncan 1996). Thus, in order to describe the flux expulsion in magnetars our model has to be modified qualitatively.
Both the investigation of flux expulsion in magnetars and the effects of accretion on the evolution of a MF permeating the whole neutron star, will be considered in forthcoming papers.

Acknowledgment

The work of D.K. was supported in part by the Russian Foundation of Basic Research under the Grant 97-02-18096(a) and by INTAS under the grant 96-0154. D.K. gratefully acknowledges the kind hospitality of the Nordic Institute for Theoretical Physics (Nordita) and of Prof. Raedler's group of Astrophysikalisches Institut Potsdam. It is also pleasure to thank Prof. C. Pethick for valuable and stimulating discussions.

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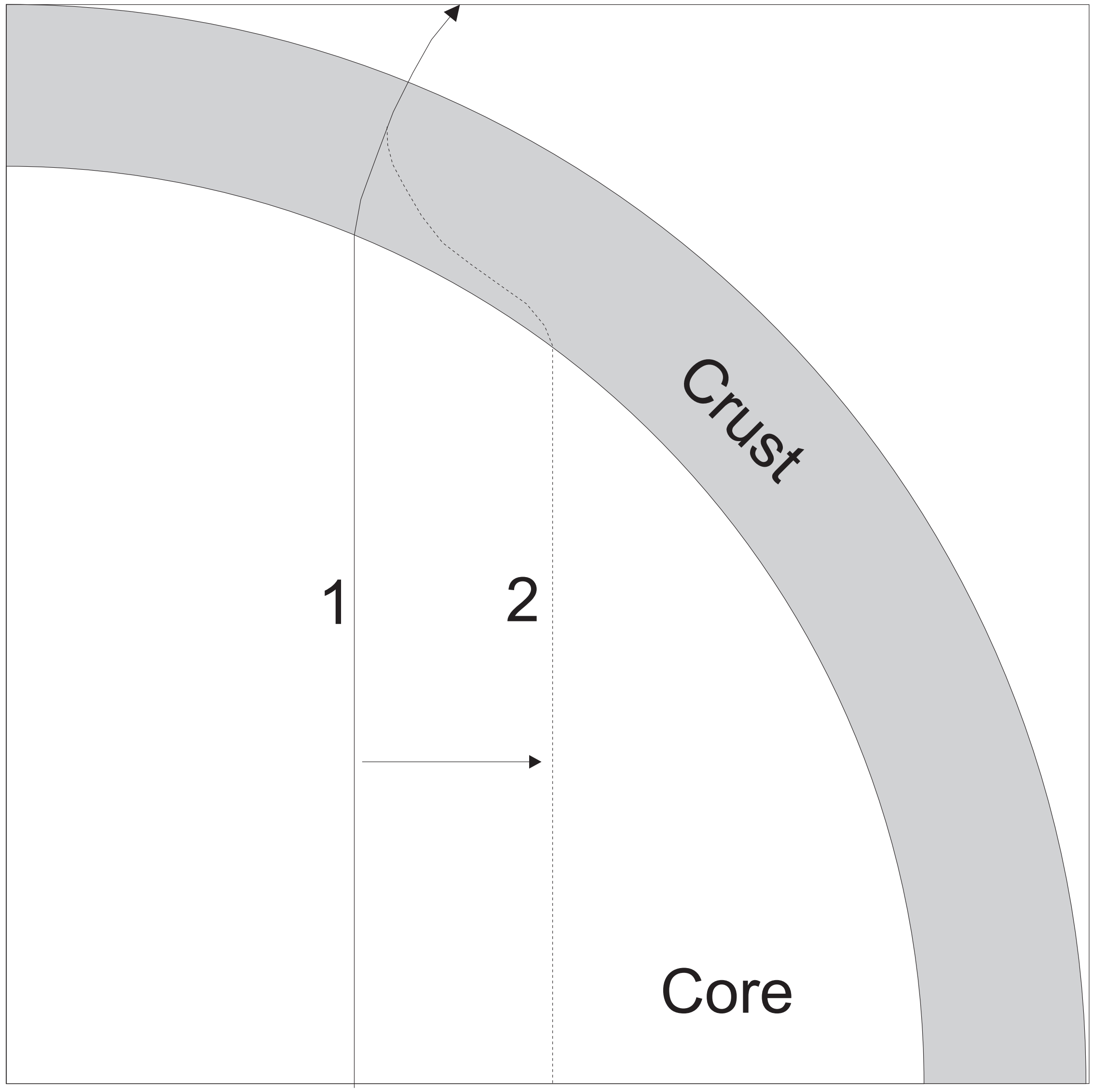
Figure captions

Fig. 1 Due to the motion of fluxoids from position 1 to position 2 the magnetic field lines are bended in the crust. This causes a change of the magnetic energy and heat release within the crust. Thus, a force raises which can counteract the movement of fluxoids.
Fig. 2 The same as in figure 1 but in terms of the normalized stream function $s(r,t)$. The gradient of s near the crust–core interface corresponds, according to equation (9), to a generation of a θ–component of the crustal magnetic field.
Fig. 3 The temporal evolution of magnetic field, forces, velocities of both kinds of tubes and spin period of the neutron star for $B_{e0} = 10^{12}$ G. Left, middle and right columns correspond to the different values of the impurity parameter: $Q = 1, 0.1, 0.01$, respectively.
Fig. 4 The same as in figure 3 but for $B_{e0} = 10^{11}$ G.
Fig. 5 The temporal evolution of the magnetic field and spin period for $B_{e0} = 10^{13}$ G. The numbers at the curves correspond to the different values of the impurity parameter Q.
Fig. 6 The temporal evolution of the surface temperature for different values of B_{e0} and Q. Curve 1: no additional heating, curve 2: $B_{e0} = 10^{11}$ G, $Q = 0.1$; curve 3: $B_{e0} = 10^{11}$ G, $Q = 0.01$; curve 4: $B_{e0} = 10^{12}$ G, $Q = 0.01$; curve 5: $B_{e0} = 10^{12}$ G, $Q = 0.1$; curve 6: $B_{e0} = 10^{13}$ G, $Q = 1$.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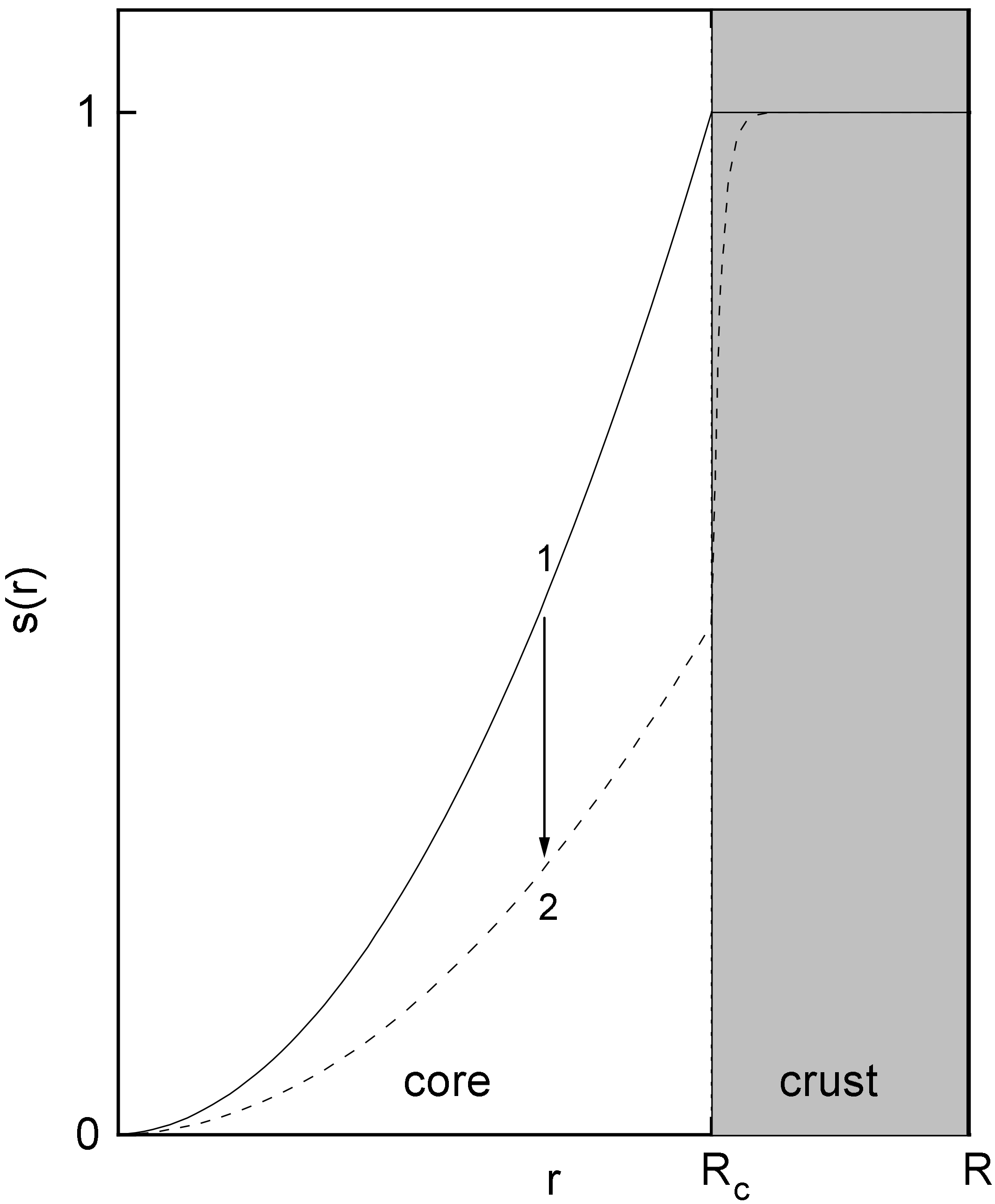
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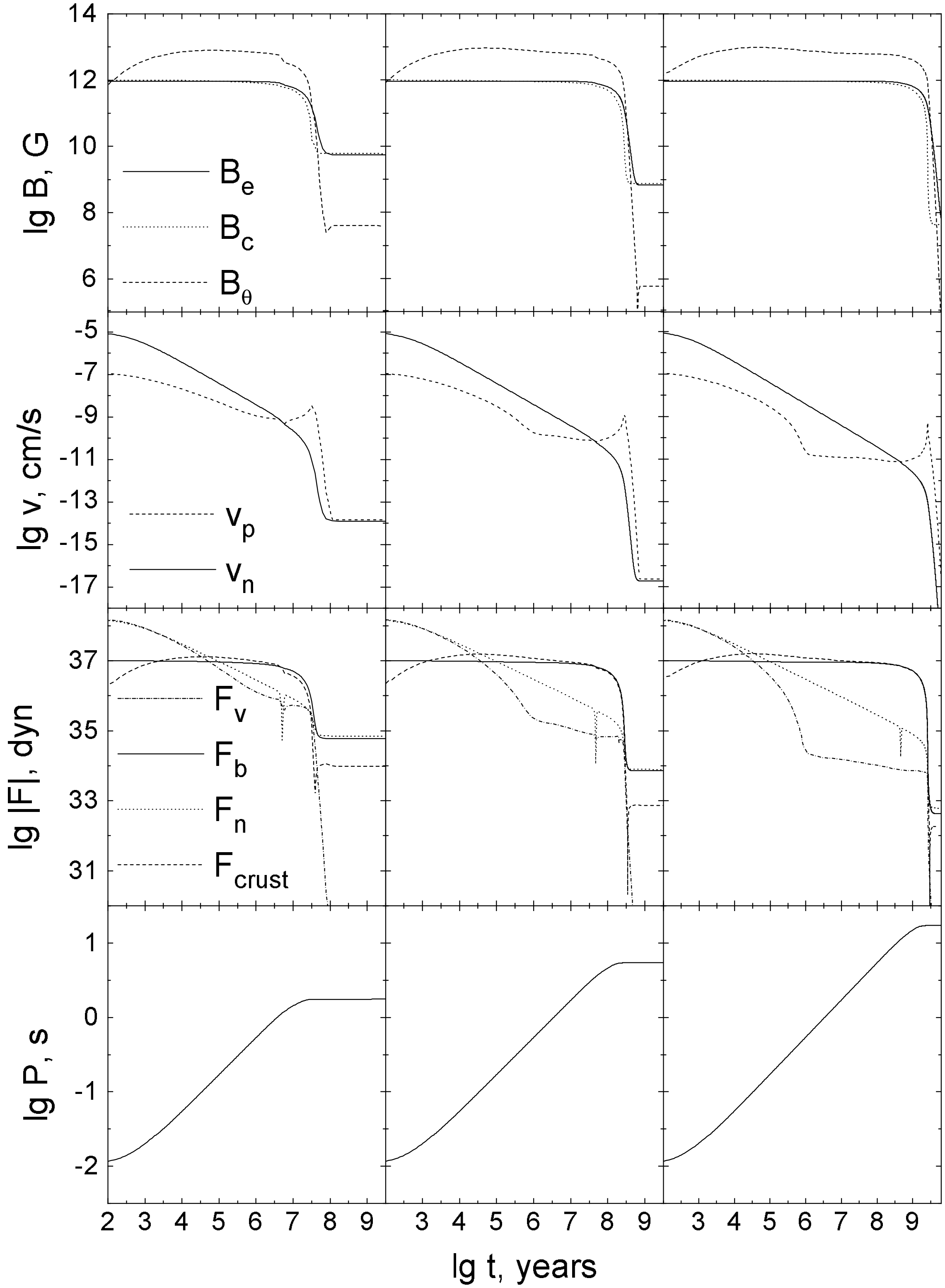

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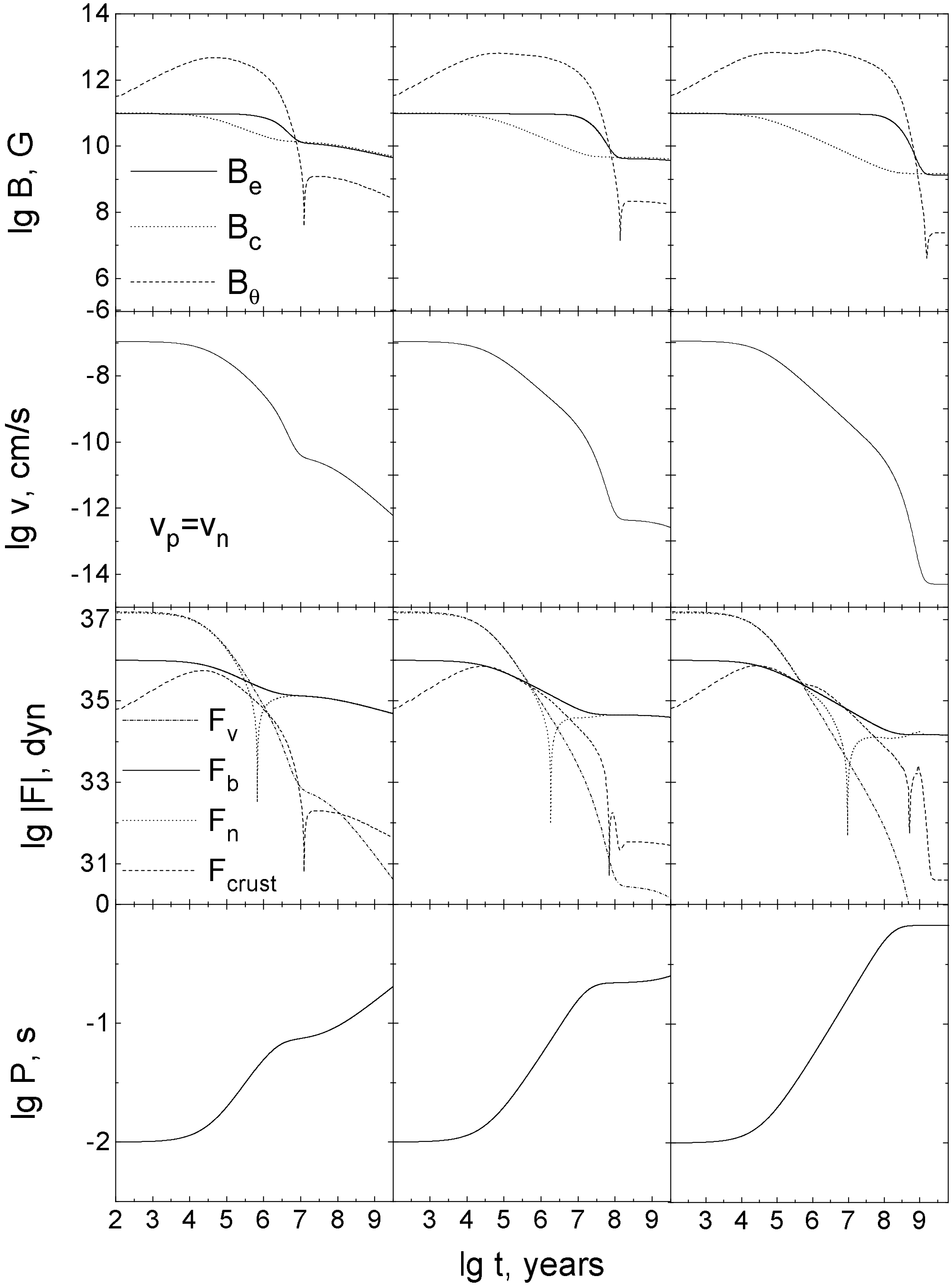
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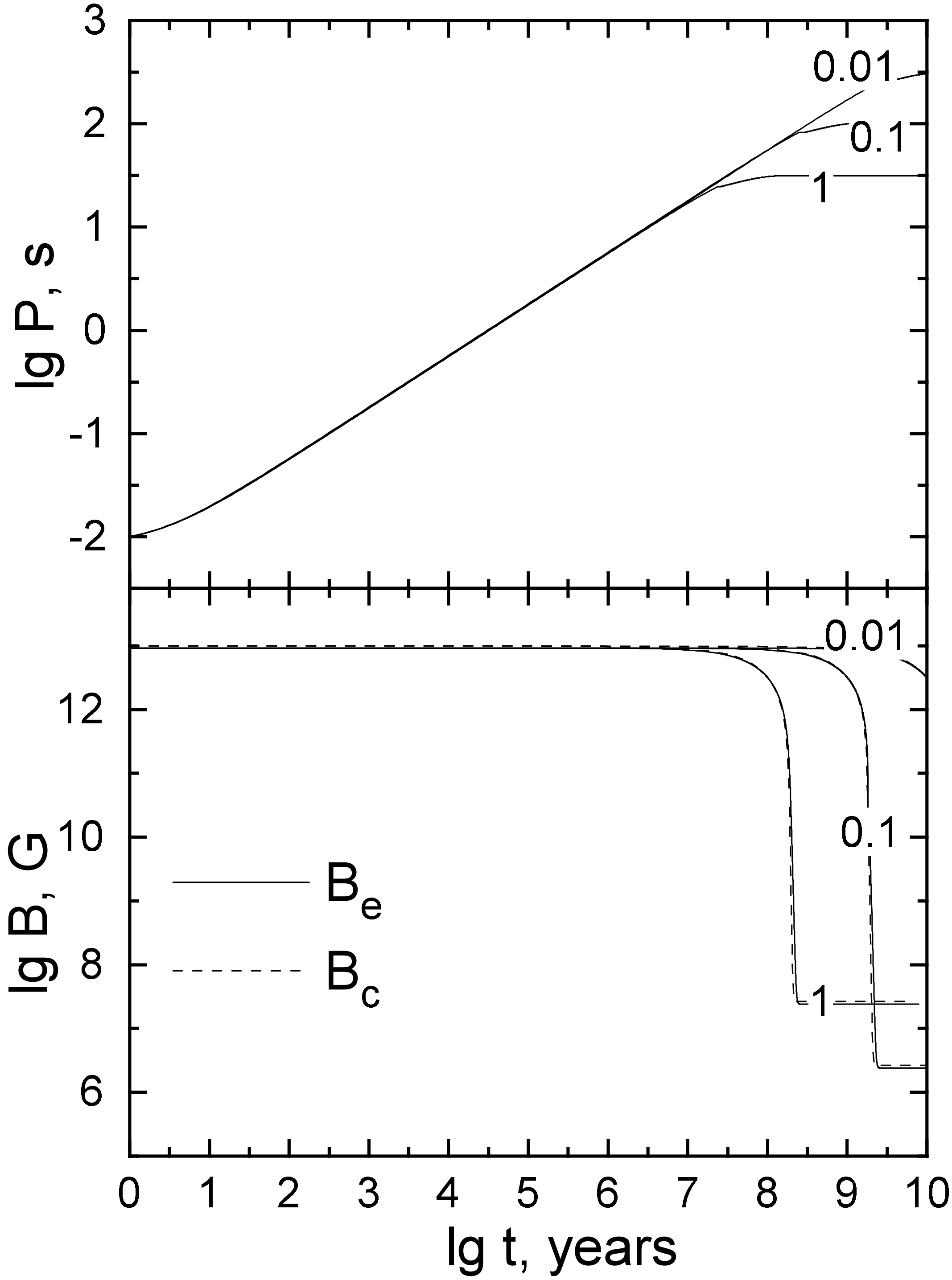

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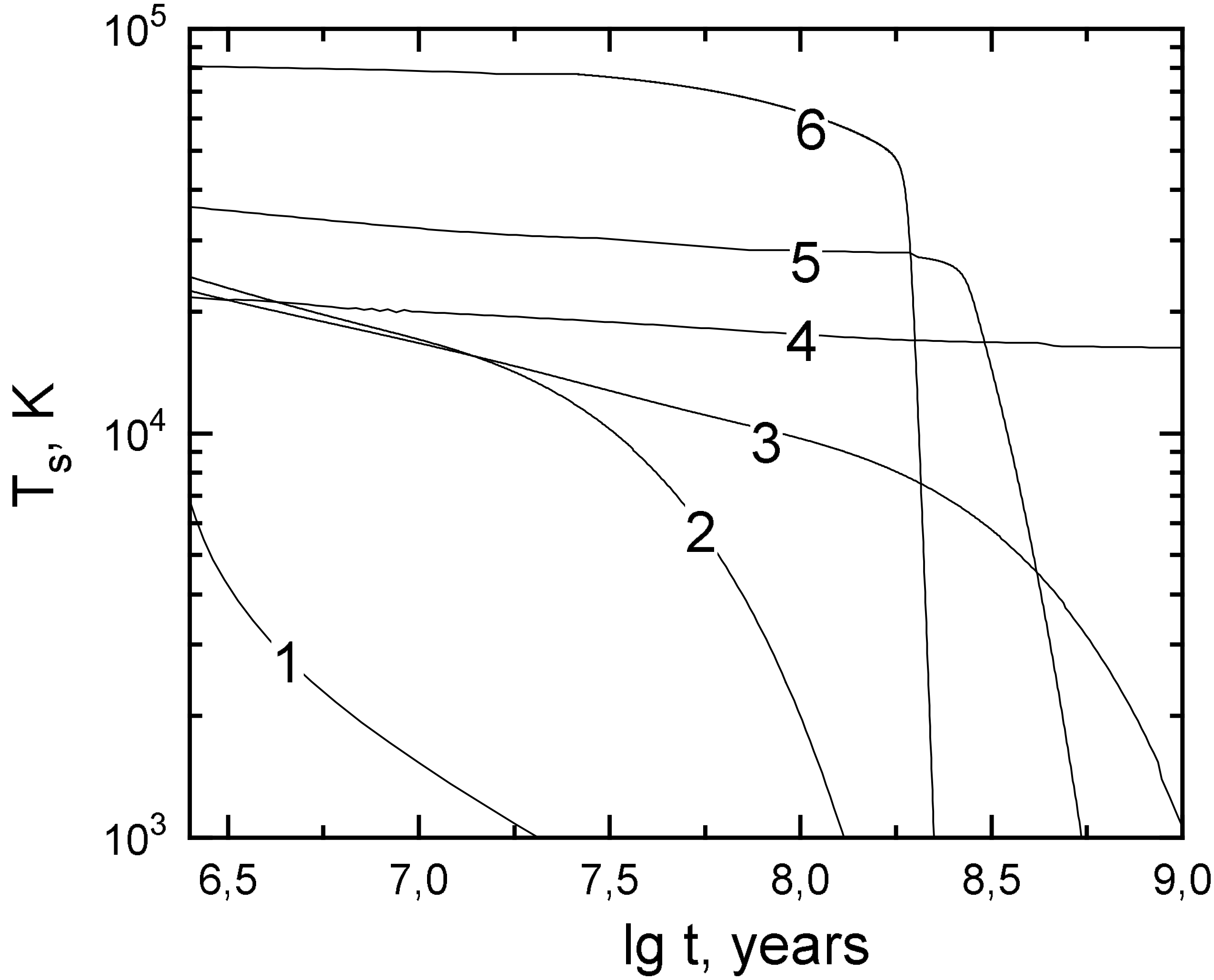

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